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Estimating the Impact of the Covid-19 Emergency on Tax Revenues in Guatemala: A Time Series Approach

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Abstract

Applications of time series models serve two different purposes: (1) as forecasting techniques, they are used to project the trajectory of a variable of interest during a certain number of future periods; (2) in the analysis of interventions, they are used to evaluate the effect of a significant disturbance on the process being studied. We use both types of application to study monthly tax revenues in Guatemala. In Section 2 we use data for 2010-2019 in order to compare two alternative models: (a) the Box-Jenkins (ARIMA) model, and (b) the Holt-Winters exponential smoothing model. In Section 3 we use post-2019 data to estimate the fiscal effects of the emergency measures implemented to contain the Covid-19 pandemic.

Keywords

Public finance, Tax revenues, Covid-19, ARIMA models, Box-Jenkins, Holt-Winters, Exponential smoothing, Time series forecasting

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1. INTRODUCTION

Time series methods have proved quite useful in a wide range of applications in business and economics. All statistical time series models are based on the analysis of historical data in a given process, in order to make projections about the future trajectory of that process. These models can be classified in two main groups: *extrapolative* models are based on the identification of trends and seasonalities in the historical data, while *auto-regressive* models are based on correlations between lagged values of the process under study. Among models of the first group, the one most commonly used in practice is the so-called Holt-Winters model (Holt, 1957; Winters, 1960; Chatfield, 1978; Roberts, 1982; Gardner and McKenzie, 1985; Chen, 1996; Chatfield, Koehler, Ord and Snyder, 2001; Goodwin, 2010; Stellwagen, 2012), while the most popular auto-regressive method is the ARIMA model, also known as the "Box-Jenkins model" (Box and Jenkins, 1970; Newbold, 1975, 1983; Harvey, 1990; Wang, 2008; Stellwagen and Tashman, 2013). There is a very extensive literature on the properties and applications of these methods, and there has been an impressive development of computational techniques in recent decades, although the basic theory that supports these methods has not changed much since they were first proposed.

Applications of these methods have basically two different purposes:

1) Forecasting techniques. In these applications the purpose is not analytic, but merely predictive, and the objective is to estimate the likely trajectory of a variable of interest during a certain number of future periods.

2) Intervention analysis. In these applications the estimated models are used to assess the impact of a significant disturbance on the process being studied. For this, the observed behavior of the variable *after* the disturbance occurs is compared with an estimate of what would have happened *in the absence of the disturbance*. In this case, the estimate is also a prediction, but it is not a forecast of future events but rather a counter-factual calculation for a period that has already elapsed.

In this paper we use both types of application to study monthly tax revenues of the central government of Guatemala. In Section 2 we analyze the data for the period 2010 to 2019, in order to compare the performance of the two models mentioned

above, and in Section 3 we use post-2019 data to estimate the fiscal effects of the emergency measures implemented to contain the Covid-19 pandemic.

2. RELATIVE PERFORMANCE OF HOLT-WINTERS AND BOX-JENKINS MODELS

Exponential Smoothing is a very popular technique in managerial applications, and one of its main characteristics is that, in generating forecasts, it gives more weight to recent history than to the more distant past. The Holt-Winters model is an exponential smoothing method that assumes that a time series has three basic components that are relatively easy to interpret: level, trend and seasonality. The level represents the current average of the series, trend indicates the average expected change, and the seasonal factors show the pattern of variation between successive periods (months, quarters, or even shorter periods). Once the current values for level, trend and seasonality have been estimated, these values are used to generate the forecasts.

Box and Jenkins (1970) popularized an alternative method which they called ARIMA ("Auto-Regressive Integrated Moving Average"). This method is similar to the Holt-Winters model, since it is also used to identify trends and seasonal patterns. However, this methodincorporates an *additional* source of information, the pattern of *auto-correlations* in the time series, which is neither a trend nor a seasonality but a sort of continuity or carryover in the effect of changes in one time period over subsequent periods. Due to the use of the information contained in these auto-correlations, the Box-Jenkins model is mathematically more complex, and therefore it is harder to interpret conceptually.

ARIMA models initially generated much enthusiasm because, under certain conditions, they generate "optimal" predictions, which means that the model's errors do not contain any information that can improve the forecasts. (Technically, such errors are described as "white noise".) However, this does not imply that ARIMA models are always better, because in practice the data do not always satisfy the required assumptions. In fact, many empirical studies show that, in spite of their theoretical superiority, ARIMA models do not always outperform other methods based on simpler foundations, and in practice the theoretical "optimality" is often not reflected in real superiority (Roberts and Harrison, 1984; Gardner, 1985, 2006;

Fildes and Makridakis, 1995; Makridakis and Hibon, 2000; Goldstein and Gigerenzer, 2009).

2.1. Empirical results

The technical aspects of both types of model are well known, and relatively detailed explanations can be found in textbooks on the subject (for example, Nelson, 1973; Goodrich, 1989; Montgomery, Johnson and Gardiner, 1990; Hamilton, 1994; Enders, 1995; Makridakis, Wheelright and Hyndman, 1997; Ord and Fildes, 2013). In this section we will compare the practical forecasting performance of these two models for the series "Tax Revenues for the Central Government of Guatemala" for a period of 10 years (2010 to 2019).[1]

Table 1: Holt-Winters and Box-Jenkins forecasts, 2010.

(A) HOLT-WINTERS					
Month	Forecast	Observed	Error	APE (%)	Error^2
Jan	3,119.0	3,240.8	121.8	3.76	14826.7
Feb	2,167.7	2,250.2	82.5	3.67	6808.2
Mar	2,961.7	2,982.2	20.5	0.69	419.7
Apr	3,274.1	3,458.7	184.6	5.34	34072.4
May	2,333.9	2,362.9	29.0	1.23	840.5
Jun	2,324.8	2,415.5	90.7	3.75	8219.2
Jul	3,723.4	3,739.3	15.9	0.43	253.9
Aug	2,333.0	2,577.6	244.6	9.49	59816.0
Sep	2,340.6	2,433.3	92.7	3.81	8589.8
Oct	3,832.2	3,670.2	-162.0	4.41	6255.7
Nov	2,481.9	2,665.4	183.5	6.89	33678.5
Dec	2,715.9	2,975.9	260.0	8.74	67608.3
Sums	33,608.3	34,772.0	1,163.7		
Yearly error: 3.35%		MAPE: 4.35%		RMSE: 147.59	

[1] Data for this study were obtained from the Guatemalan central bank webpage (*see* https://www.banguat.gob.gt/es/page/tributarios, this site indicates the Ministry of Public Finance as the primary source). The basic dataset is available from the author upon request.

(B) BOX-JENKINS					
Month	Forecast	Observed	Error	APE (%)	Error^2
Jan	3,184.7	3,240.8	56.1	1.73	3147.4
Feb	2,217.9	2,250.2	32.3	1.43	1040.4
Mar	2,985.5	2,982.2	-3.3	0.11	10.6
Apr	3,402.9	3,458.7	55.8	1.61	3110.0
May	2,506.1	2,362.9	-143.2	6.06	20501.1
Jun	2,387.1	2,415.5	28.4	1.18	807.5
Jul	3,805.8	3,739.3	-66.5	1.78	4426.9
Aug	2,435.3	2,577.6	142.4	5.52	20263.5
Sep	2,434.1	2,433.3	-0.8	0.03	0.7
Oct	3,939.6	3,670.2	-269.4	7.34	72554.8
Nov	2,605.6	2,665.4	59.8	2.24	3571.6
Dec	2,801.8	2,975.9	174.1	5.85	30301.8
Sums	34,706.4	34,772.0	65.6		
Yearly error: 0.19%		MAPE: 2.9%		RMSE: 115.37	

To illustrate the performance statistics, detailed calculations for the year 2010 are shown in Table 1, both for the Holt-Winters model (Panel A) as well as the Box-Jenkins model (Panel B). Forecasts for the twelve months of 2010 were generated from models calculated using monthly data from 1995 to 2009. The Holt-Winters is an exponential smoothing model with multiplicative seasonality, and the Box-Jenkins is an ARIMA $(0,1,1)(0,1,1)_{12}$ model with logarithmic transformation.[2]

The first column in each panel shows the respective months, and the following columns show the forecasted values from each model, the actual value in each month, and the corresponding errors. The sums show the yearly totals for each column, and here we can see that the Box-Jenkins model performed much better

[2] The models were estimated with the "Forecast Pro" (4.1) software program, developed by Business Forecast Systems, Inc. (Belmont, Massachusetts, USA). Identification of the ARIMA model was done by the program.

than the Holt-Winters in forecasting the yearly total: total tax revenues in Guatemala were 34,772 million quetzals in 2010, so the Holt-Winters forecast of 33,608.3 million was off by 1,163.7 million (an error of 3.35%), whereas the Box-Jenkins forecast of 34,706.4 million had a much smaller error: 65.6 million quetzals (0.19%).

The column labeled APE ("Absolute Percentage Error") shows the absolute values of the monthly percentage errors. The average of these percentage errors, known as MAPE ("Mean Absolute Percentage Error"), is used as a comparative measure of the accuracy of the monthly forecasts. Here we can see that, although the Box-Jenkins forecast did not always outperform in each individual month (in three of the months the percentage error was smaller for the Holt-Winters), the Box-Jenkins model did show the best overall result: on average, the percentage error in the monthly forecasts was 2.91%, in absolute terms, compared to 4.35% for the Holt-Winters.

The last column in each panel shows, for each month, the squared error. The square root of the average of these squared errors, known as RMSE ("Root Mean Squared Error"), is also used as a complementary measure of forecast accuracy, and can be interpreted as the standard deviation of the monthly errors. The Box-Jenkins model shows the best result in terms of this statistic as well: 115.4 million quetzals/month, compared to 147.6 million/month for the Holt-Winters.

This analysis was repeated for all subsequent years, from 2011 to 2019. For each year the two models were estimated using the data from January 1995 to December of the year prior to the forecasted year. The twelve monthly forecasts were then computed, and for each year a comparative analysis similar to the one in Table 1 was performed. Table 2 summarizes the results of these comparisons.

Table 2: Relative Performance of Box-Jenkins and Holt-Winters models, 2010-2019.

	Observed	Forecast		Yearly error (%)		MAPE (monthly)		RMSE (monthly)	
		B-J	H-W	B-J	H-W	B-J	H-W	B-J	H-W
2010	34,772.0	34,706.4	33,608.3	0.19	3.35	2.91	4.35	115.4	147.6
2011	40,292.2	37,128.6	36,819.2	7.85	8.62	7.69	8.49	367.6	381.3
2012	42,820.0	42,892.4	42,327.3	-0.17	1.15	2.67	2.41	115.8	102.6
2013	46,335.5	46,435.1	45,277.7	-0.22	2.28	5.21	5.54	236.4	271.2
2014	49,096.9	49,670.8	48,286.0	-1.17	1.65	3.38	3.39	183.2	196.3
2015	49,730.7	52,878.4	51,843.7	-6.33	-4.25	6.64	4.52	292.7	206.6
2016	54,109.6	53,002.3	51,849.0	2.05	4.18	4.89	5.50	277.7	319.3
2017	56,684.1	57,648.3	57,503.8	-1.70	-1.45	4.19	4.37	246.4	262.3
2018	58,835.7	60,251.3	59,315.2	-2.41	-0.82	3.35	2.79	182.8	153.3
2019	62,593.6	62,107.3	61,257.1	0.78	2.14	2.66	3.11	192.6	231.4
Averages				2.29	2.99	4.36	4.45	221.06	227.19

Although the Box-Jenkins model outperformed in 2010, this was not always the case in subsequent years. In 2011 there was an unexpected rise in tax revenues, and neither of the two models was able to forecast this accurately. In the following years, the average error was generally quite small for both models, and in three of those years the Holt-Winters forecast outperformed the Box-Jenkins.

Overall, the average yearly percentage error was slightly better for the Box-Jenkins, although the difference was not very large: 2.29% versus 2.99%.[3] For the other performance statistics, the results for the two models were very similar: in terms of monthly MAPE, the average was 4.36% for the Box-Jenkins and 4.45% for the Holt-Winters, and the RMSE's were also, on average, practically the same for both models (221.06 million quetzals/month and 227.19 million/month, respectively).[4]

[3] These averages were calculated using the absolute values of the yearly percentage errors.

[4] This analysis was done in terms of nominal quetzals (i.e., with no adjustment for inflation). In principle it might be supposed that an analysis in terms of constant quetzals would give better results, though in fact that was not the case. The analysis summarized in Table 2 was repeated using tax revenues deflated by the monthly CPI. The results (not shown due to space limitations) indicate

2.2. Stability of the underlying process

Time series models work reasonably well when we can assume a certain degree of continuity between past and future, i.e., that the regularities observed in the past reflect a certain stability in the process under study, which in turn justifies the assumption that these regularities will also be observed in the future, at least in the short run. In the case of tax revenues in Guatemala this assumption seems valid, in part owing to the relative efficacy of the forecasts (as shown by Table 2), but also due to certain characteristics of the estimated models.

that the models estimated using deflated data do not perform better than the models for nominal data. This might be due to measurement errors in the inflation estimates: it is possible that, by deflating the nominal data with the CPI, we are introducing a certain amount of "noise" into the analysis. For the rest of this paper we will limit our comments to the results using nominal data.

Table 3: Seasonal Factors in the Holt-Winters models.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2010	1.161	0.803	1.093	1.203	0.854	0.847	1.351	0.843	0.842	1.373	0.886	0.965
2011	1.168	0.807	1.080	1.216	0.846	0.847	1.330	0.867	0.839	1.318	0.898	0.984
2012	1.150	0.811	1.201	1.207	0.854	0.873	1.287	0.876	0.843	1.280	0.871	0.936
2013	1.142	0.822	1.192	1.181	0.864	0.898	1.277	0.874	0.829	1.284	0.869	0.945
2014	1.183	0.820	1.157	1.227	0.886	0.852	1.322	0.869	0.828	1.302	0.860	0.903
2015	1.192	0.811	1.104	1.257	0.892	0.852	1.354	0.877	0.843	1.290	0.850	0.895
2016	1.189	0.817	1.071	1.262	0.882	0.862	1.386	0.873	0.842	1.299	0.852	0.890
2017	1.170	0.799	1.029	1.250	0.951	0.862	1.393	0.894	0.824	1.271	0.859	0.904
2018	1.162	0.796	1.049	1.247	0.905	0.924	1.366	0.881	0.817	1.281	0.873	0.901
2019	1.184	0.795	1.029	1.256	0.900	0.904	1.372	0.877	0.815	1.304	0.889	0.888
Averages	1.170	0.808	1.100	1.231	0.884	0.872	1.344	0.873	0.832	1.300	0.871	0.921
Std. Dev.	0.017	0.010	**0.063**	0.028	0.031	0.027	0.039	0.013	0.011	0.029	0.016	0.034

An interesting aspect of the Holt-Winters model is reflected in the *seasonal factors*, a key component of the model's structure. Apart from their role in the computation of the monthly forecasts, the seasonal factors are interesting in their own right since they facilitate the interpretation of the model. The estimated seasonal factors for each of the forecast years are shown in Table 3. The first row indicates, for instance, that for the year 2010 it was estimated that tax revenues for January would be a little over 16% above the yearly average, whereas revenues for February would be almost

20% *below* the yearly average, in March a little over 9% above, and so on. Since the Holt-Winters method gives more weight to observations from the most recent past, and since each year the model was updated by adding the historical data for the year before, this means that the seasonal factors for each successive year would tend to be quite variable if the underlying process were very unstable. On the other hand, if the process is relatively stable then the estimated seasonal factors should not change much from year to year. Table 3 shows that the estimated seasonal factors are in fact quite stable: March is the only month that shows some degree of variability in its seasonal factors.[5]

Table 4: Coefficients of the ARIMA (0,1,1)(0,1,1)12 models.

	b[1]	B[12]
2010	0.9066	0.6497
2011	0.9078	0.6425
2012	0.9064	0.6440
2013	0.9068	0.6402
2014	0.9069	0.6458
2015	0.9074	0.6506
2016	0.9019	0.6481
2017	0.9030	0.6502
2018	0.9032	0.6475
2019	0.9025	0.6472
Average	0.9053	0.6466
Std. Dev.	0.0023	0.0035

Due to its greater mathematical complexity, the parameters of the Box-Jenkins models are not as easy to interpret but, in general, if a process is highly unstable it is often the case that when new observations are added to the sample the estimated parameters tend to be highly variable, and the automatic algorithms often generate ARIMA models with very different specifications. In this case, however, for every

[5] The standard deviation of the seasonal factors for that month (highlighted in bold type) is almost three times larger than the average for the other 11 months. This must have some explanation, but it is not something that can be determined from the time series alone.

year the algorithm generated the same ARIMA (0,1,1)(0,1,1)12 model, which depends upon two coefficients, known as b[1] and B[1]. Table 4 shows the values of these coefficients for the yearly models, and it can be observed that the estimated coefficients do not vary much from one year to another, which tends to support the assumption that the underlying process is relatively stable.

3. FISCAL IMPACT OF THE COVID-19 PANDEMIC

The global Covid-19 pandemic had a very great impact, worldwide, in terms of mortality and public health, and also in economic terms due to the restrictions implemented to contain the spread of the virus. One noticeable effect of these restrictions was a sudden reduction in tax revenues in practically every country in the world, and Guatemala was no exception: yearly tax revenues dropped from 62,593.6 million quetzals in 2019 to 60, 279.4 million in 2020, a 3.7% reduction.[6]

It would be incorrect, however, to take this as a measure of the *total* effect of the Covid-19 restrictions, because this is not the relevant comparison. Tax revenues for 2020 should not be compared to revenues for the previous year, but rather with an estimate of what they *would have been* in 2020 in the absence of the pandemic.

[6] During the period covered by this study, the only other drop in yearly tax revenues occurred in 2009
(-4.6%), which most likely was due to the effects in Guatemala of the 2008-2009 world recession.

Table 5: Holt-Winters and Box-Jenkins forecasts, Jan 2020-June 2021.

2020	Observed	Forecast		Difference		Difference (%)	
		B-J	H-W	B-J	H-W	B-J	H-W
Jan	6,587.4	6,295.8	6,279.2	291.5	308.2	4.6	4.9
Feb	4,261.0	4,213.5	4,173.9	47.5	87.1	1.1	2.1
Mar	3,653.7	5,484.3	5,423.1	-1,830.6	-1,769.3	-33.4	-32.6
Apr	6,953.1	6,670.3	6,641.2	282.8	311.8	4.2	4.7
May	3,477.9	4,850.0	4,830.9	-1,372.2	-1,353.0	-28.3	-28.0
Jun	3,532.0	4,724.0	4,687.3	-1,192.0	-1,155.4	-25.2	-24.6
Jul	6,154.3	7,445.1	7,419.0	-1,290.8	-1,264.7	-17.3	-17.0
Aug	3,934.3	4,770.7	4,737.2	-836.4	-802.9	-17.5	-16.9
Sep	4,255.6	4,491.4	4,457.6	-235.8	-202.0	-5.3	-4.5
Oct	7,592.8	7,284.7	7,268.7	308.1	324.0	4.2	4.5
Nov	4,731.0	4,894.5	4,851.9	-163.5	-120.9	-3.3	-2.5
Dec	5,146.3	5,063.5	5,010.0	82.9	136.3	1.6	2.7
Sums	60,279.4	66,187.9	65,780.0	-5,908.5	-5,500.6		
		Estimated reduction:		-8.93%	-8.36%		

2021	Observed	Forecast		Difference		Difference (%)	
		B-J	H-W	B-J	H-W	B-J	B-J
Jan	6,893.7	6,652.3	6,546.4	241.4	347.3	3.6	5.3
Feb	4,730.9	4,452.0	4,350.9	278.9	380.0	6.3	8.7
Mar	6,255.2	5,794.8	5,652.2	460.5	603.0	7.9	10.7
Apr	7,775.0	7,047.9	6,920.8	727.1	854.2	10.3	12.3
May	5,258.4	5,124.6	5,033.6	133.8	224.9	2.6	4.5
Jun	5,106.0	4,991.4	4,883.3	114.6	222.7	2.3	4.6

Figure 1: Tax revenues in Guatemala, Jan 2020-June 2021, observed and forecasted values

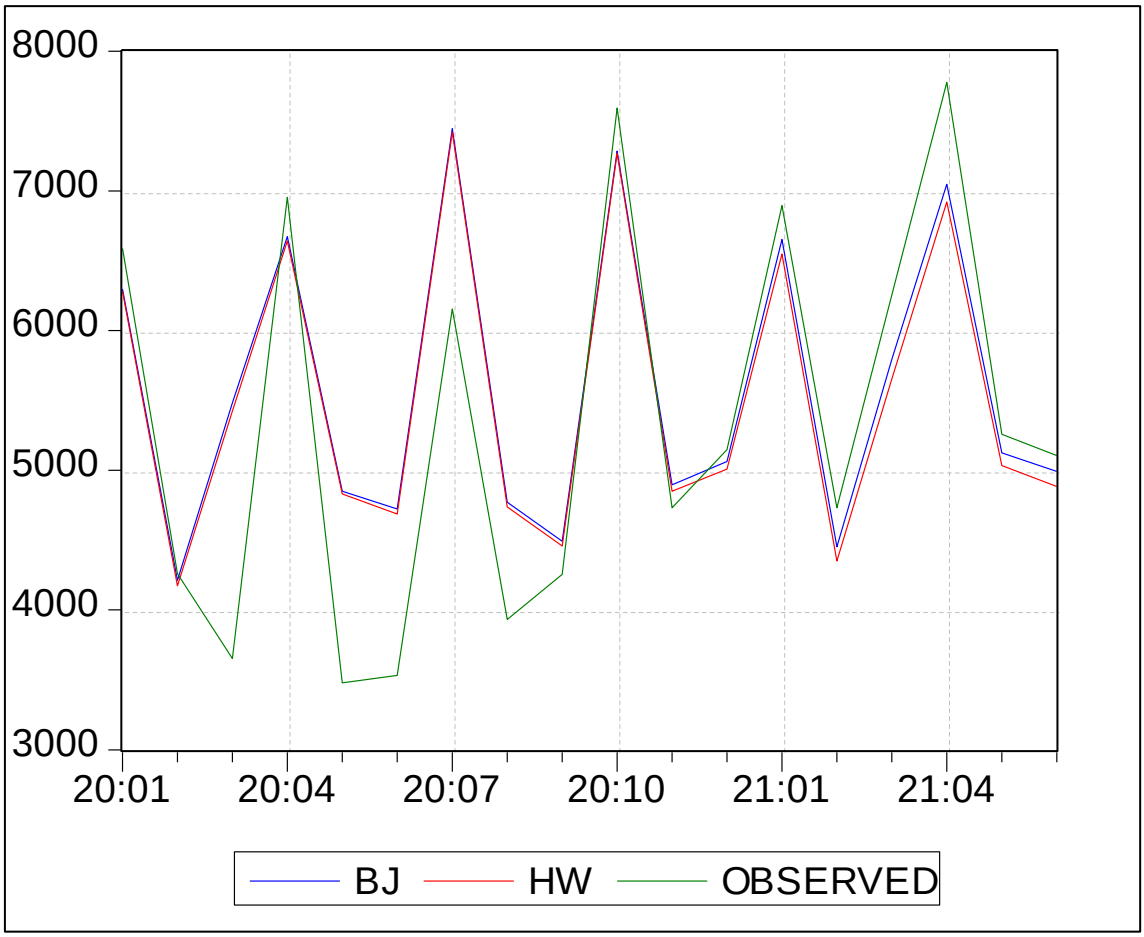

Table 5 and Figure 1 compare the forecasts for both models, with estimates from data up to December 2019, with actual tax revenues in each month of 2020 and the first six months of 2021. The Covid-19 pandemic was of course completely unpredictable, and no forecasting method would have anticipated an event of this kind. Therefore, it is no surprise that both models fail as *ex ante* forecasts. On the other hand, since we know that both models performed quite well in the previous years, we can take the forecasts for 2020 and 2021 as a reasonable estimate of expected tax revenues under *normal* conditions, and the difference between expected and observed tax revenues can be taken as an *ex post* estimate of the impact of the Covid-19 emergency measures.[7]

[7] It is possible that other factors also changed, but for 2020 it is hard to think of any other factor that might have had an impact comparable to that of the Covid-19 pandemic.

Table 5 and Figure 1 clearly show that both models generate essentially the same forecasts. In practice, it appears that both models are equivalent characterizations of the same underlying process. If we compare the yearly totals, the drop in tax revenues in 2020 was a little over 5,900 million quetzals (-8.93%) according to the Box-Jenkins forecast, and 5,500 million quetzals (-8.36%), according to the Holt-Winters forecast.

If we examine the data month by month, we can see that in January and February of 2020 tax revenues were in fact slightly above the expected level, although the difference is within the expected margin of error for these models. In Guatemala the Covid-19 emergency measures were implemented in March 2020, and that month shows a very sharp reduction (over 30% for both models). In April there was a recovery, but then in May, June, July and August tax revenues were much lower than expected. In September revenues were also lower than expected, but the difference was within the margin of error. It seems safe to conclude that, after a strong downturn that lasted several months, tax revenues in Guatemala more or less normalized from September 2020 onwards. By October of that year the monthly values had practically recovered their expected levels (and by 2021 slightly exceeded them).

4. CONCLUSIONS

Forecasts of future tax revenues are obviously an important element for economic policy-making, and in this study we have shown that, in the case of Guatemala, forecasting techniques based on time series models can make a useful contribution in this regard.

To be sure, the accuracy of such forecasts is never guaranteed, and errors will always be present. If the underlying processes are relatively stable, the margins of error will not be too large, and this is what we generally observe in the case of tax revenues in Guatemala. On the other hand, sometimes genuine surprises occur, such as the recent Covid-19 pandemic and its socio-economic repercussions.

Surprises, by definition, are unpredictable, and no purely statistical model is capable of anticipating events of that sort. On the other hand, what these models *can* provide is a benchmark for evaluating the *effects* of large disturbances on the process

under study. In this paper we used the forecasts for 2020 as a "control" to estimate the likely trajectory of tax revenues in Guatemala *if the pandemic had not occurred*. With this metric, we estimated that the reduction of yearly tax revenues during 2020 as a result of the Covid-19 emergency was about 9% according to the Box-Jenkins model, and a little less than that (8.4%) according to the Holt-Winters model. Obviously, we cannot really know if these estimates are accurate or not, but probably they are closer to the truth than simply assuming that the entire fiscal effect of the pandemic is reflected in the drop in yearly revenues from 2019 to 2020. The fiscal effect of the pandemic seems to have petered out by September-October of 2020, and the more recent figures indicate that monthly tax revenues have recovered (and in recent months actually exceeded) their pre-pandemic expected levels.

A topic for further exploration in this field is the application of these techniques to different sources of tax revenue. It is quite likely, for instance, that different kinds of taxes will have different seasonalities in their collection, and the revenues from some taxes might be more volatile or unstable than others. Also, some models are probably more adaptable to certain kinds of taxes than to others, and as regards the effects of the Covid-19 restrictions, they might well have been greater for some types of taxes than for others. There is scope, therefore, for further research in this area.

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